18.06 (Fall '12) Problem Set 6

This problem set is due Thursday, October 25, 2012 by 4pm in 2-255. The problems are out of the 4th edition of the textbook. For computational problems, please include a printout of the code with the problem set (for MATLAB in particular, `diary(''filename'')` will start a transcript session, `diary off` will end one, also copy and paste usually work as well.)

1. Do Problem 9 from 4.4. (You can use a computer if you want.)

2. Do Problem 23 from 4.4.

3. Do Problem 31 from 4.4.

4. Do Problem 3 from 8.5

5. Do Problem 12 from 8.5.

6. Whether you are familiar with probability or not, there is a certain kind of variable called a standard normal distribution. In MATLAB you can get one million samples by typing `x=randn(1e6,1)`. You can compute averages of polynomials in x, for example, the average value of $2 * x^3 + x^2 + x + 5$ is `mean(2*x.^3+x.^2+x+5)`. (You need the dot for raising a vector to a power or multiplying two vectors pointwise). These averages are approximate because they are based on random samples. Convince yourself that `h1=x`, and `h2=x.^2-1`, and `h3=x.^3-3*x` are orthogonal by typing `mean(h1.*h2)`, etc. and getting a number that has at least two zeros after the decimal point. What is the length of h1,h2, and h3? (`mean(h1.*h1)`, etc.)

7. Do Problem 3 from 5.1.

8. Do Problem 12 from 5.1.

9. Do Problem 25 from 5.1.

10. Let M be a 6 by 6 matrix whose entries are all 1 or -1. We will explore the determinants. In MATLAB or equivalent, compute `det(sign(randn(6)))` enough times that you believe that the determinant is always divisible by 32.

    Explain why the determinant is divisible by 32. (Hint. The first step of elimination is going to give you a matrix with ± 1 in the top left corner and a 5 by 5 matrix whose entries are -2, 2 or 0, you can then try to explain why such a 5 by 5 matrix has its determinant divisible by 32.).

18.06 Wisdom: Really understand the determinant and think about how it relates to rank and invertibility. Learn to stereotype a matrix by whether its determinant is 0 or not (what do these two situations mean?) Think about what the determinant is (and what it means) for special matrices we've seen, such as the identity matrix, permutation matrices, elimination matrices, Q (or Q^T).